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Computational Aspects of q-Method

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Abstract : *This paper contains some definitions, abbreviations, acronyms, concepts and coding of Numerical Computation involving q method. Numerical Computation plays an imperative role in solving real time and real life problems of engineering, mathematics and physics. It is an approach for solving complex mathematical problems using arithmetic operations. This approach involves formulation of mathematical models of physical situations that can be solved with mathematical operations. Applications of computer oriented numerical methods have become an integral part of life of all the modern scientists.*

Keywords :*q method, q hyper-geometric function, basic hyper-geometric function etc.*

I. Introduction

The field of numerical analysis predates the discovery of modern computers by many centuries. Linear interpolation was already in use for more than ten centuries. Many great mathematicians of the past were engrossed in study of numerical analysis and it is alsoapparent from the names of important algorithms like Newton's method, Lagrange interpolation polynomial, Gaussian elimination, or Euler's method.To facilitate computations by hand, large books were produced with formulas and tables of data such as interpolation points and function coefficients. Using these tables, often calculated out to sixteen decimal places or more for some functions, one could look up values to plug into the formulas given and achieve very good numerical estimates of some functions.

The mechanical calculator was also developed as anapparatus for hand computation. These calculators evolved into electronic computers in the first generation of computers, and it was then found that these computers were also useful for administrative purposes. The discovery of the computer also influenced the field of numerical analysis, since now longer and more complicated calculations could be done. Numerical computing with the help of some special functions enhanced computational techniques.

Heine's **basic hypergeometric series**, or **hypergeometric *q*-series**, are q-analogue generalizations of generalized hypergeometric series, and are in turn generalized by elliptic hypergeometric series. A series x_n is called hypergeometric if the ratio of successive terms x_{n+1}/x_n is a rational function of n. If the ratio of successive terms is a rational function of q^n, then the series is called a basic hypergeometric series. The number q is called the base.The basic hypergeometric series was first considered by Eduard Heine (1846). It becomes the hypergeometric series $F(\alpha,\beta;\gamma;x)$ in the limit when the base q is 1.Some of the q analogues are explained below.

1.1 q Analogue of Exponential Function

***q*-exponential**is a *basic* analogue [Exton(1983)] of the exponential function, namely the eigen function of a q-derivative. There are many q-derivatives, for example, the classical q-derivative, the Askey-Wilson [Wilson et al. (1985)] operator, etc. Therefore, unlike the classical exponentials, q-exponentials are not unique. Three variants of exponential functions are given below. Third one is generalized formula and we can get first and second by changing values of α.

$$E_{q^{-1}}(x) = \sum_{r=0}^{\infty} \frac{x^r q^{r(r-1)/2}}{[r;q]!} \tag{1.1}$$

$$E_q(x) = \sum_{r=0}^{\infty} \frac{x^r}{[r;q]!} \tag{1.2}$$

$$E(q,\alpha;x) = \sum_{r=0}^{\infty} \frac{x^r q^{\frac{r\alpha(r-1)}{2}}}{[r;q]!} \tag{1.3}$$

1.2 *q Analogue of Basic*Integration: The inverse operation to basic differentiation has also been discussed at some length by F.H. Jackson [Jackson (1904)].This is represented by the symbol $\int_a^b \emptyset(x)\, d\,(qx)$ and is referred to as q-integration or basic integration. When q tends to unity, the basic integral reduces to the ordinary integral. The operations of basic differentiation and integration correspond exactly in every way to ordinary differentiation and integration of which these are generalizations.

$$\int_a^b f(x)d(qx) = (1-q)\{b\sum_{r=0}^{\infty} q^r f(q^r b) - a\sum_{r=0}^{\infty} q^r f(q^r a)\} \tag{1.4}$$

$$\int_0^c f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^r f(q^r c)\} \tag{1.5}$$

$$\int_{cq}^{\infty} f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^{r+1} f(q^{r+1} c)\} \tag{1.6}$$

$$\int_0^{\infty} f(x)d(q,x) = (1-q)\sum_{i=-\infty}^{\infty} q^i f(q^i) \tag{1.7}$$

1.3 q Analogue of Trigonometric Functions : As like exponential function, various analogues of circular functions have been introduced. Jackson gave a class of q-circular function[Exton (1983)], [Gasper et al. (2004)] .He also introduced a class of circular function from point of view of pseudo-periodicity.

$$sin_q(x) = x\sum_{r=0}^{\infty} \frac{(-x^2)^r}{[2r+1;q]!} = x0F1(-;\tfrac{3}{2};q^2;-\left[\tfrac{1}{2};q^2\right]^2 x^2) \tag{1.8}$$

$$cos_q(x) = \sum_{r=0}^{\infty} \frac{(-x^2)^r}{[2r;q]!} = 0F1(-;\tfrac{1}{2};q^2;-\left[\tfrac{1}{2};q^2\right]^2 x^2) \tag{1.9}$$

1.4 Properties of Trigonometric Functions

$$sin_q(x)sin_{1/q}(x) + cos_q(x)cos_{1/q}(x) = 1 \tag{1.10}$$

$$cos_q(x)cos_{1/q}(x) - sin_q(x)sin_{1/q}(x) = cos_q(2x) \tag{1.11}$$

1.5 Basic Differentiation operator: Jackson introduced the operative symbol[Exton (1983)], [Gasper et al. (2004)] for basic differentiation defined by the relation $\Delta\{\emptyset\,(x)\} = \{(x) - (qx)\}\, x^{-1}\,(1-q)^{-1}$, The operation of basic differentiation is defined by [Jackson (1904)] the relations

$$B_{q,x}\emptyset(x) = \frac{\emptyset(x)-\emptyset(qx)}{x(1-q)} = \sum_{r=0}^{\infty} \frac{(q-1)^r x^r d^{r+1}\emptyset(x)}{(r+1)!\, dx^{r+1}}, \tag{1.12}$$

, where x and q may be real or complex. This becomes the same as ordinary differentiation as the base q tends to unity. In order to avoid the possibility of perplexity with the ordinary difference operator, we shall write $B_{q,x}$ instead of Δ. Furthermore, the subscripts q and x will be omitted provided that there is no chance of vagueness. It will be seen that the possibility now arises of the existence of certain types of difference equations based upon this operator.

$$D_{q,x}f(x) = \frac{f(qx)-f(x)}{x(q-1)} \tag{1.13}$$

1.6 Basic analogue of Taylor's Theorem : Jackson introduced [Exton (1983)] q analogue of Taylor's Theorem

$f(x) = f(a) + \frac{(x-a)^{(1)}}{[1;q]} D_q f(a) + \frac{(x-a)^{(2)}}{[2;q]!} D_q^2 f(a) + \ldots + \frac{(x-a)^{(n)}}{[n;q]!} D_q^n f(a)$, where

$R_n = \frac{(x-a)^{(n+1)}}{[n+1;q]!} D^{(n+1)} f(\xi)$, where ξ lies between x and a. (1.14)

1.7 Variants of Laplace Transform : Hahn [Hahn (1949)] defined two analogues of Laplace Transform by the help of the integral equations

$$L_{q,s}f(x) = \frac{1}{1-q}\int_0^{\frac{1}{s}} E_q(qsx) f(x) d(q,x) \tag{1.15}$$

$$\mathcal{L}_{q,s}f(x) = \frac{1}{1-q}\int_0^{\infty} e_q(-sx)f(x)d(q,x) \tag{1.16}$$

$\mathrm{R1}(s) \geq 0$

$$L_{q,s} = \frac{1}{1-q}\int_0^{\frac{1}{s}} E_q\,(qsx)f(x)d(q,x) = \frac{(q,\infty)}{s}\sum_{j=0}^{\infty}\frac{q^j f(s^{-1}q^j)}{(q;j)} \tag{1.17}$$

$$\mathcal{L}_{q,s}f(x) = \frac{1}{1-q}\int_0^{\infty} E_q(-sx)f(x)d(q,x) = \frac{1}{\prod_{n=0}^{\infty}(1+sq^n)}\sum_{j=-\infty}^{\infty} q^j f(q^j)(1+s)_j \tag{1.18}$$

1.8 Basic Analogue of Heine's Series : This series can be represented as [Heine (1847)], [Exton (1983)], [Gasper et al. (2004)]

$$1+\frac{(1-q^a)(1-q^b)}{(1-q^c)(1-q)}x + \frac{(1-q^a)(1-q^{a+1})(1-q^b)(1-q^{b+1})}{(1-q^c)(1-q^{c+1})(1-q)(1-q^2)}x^2 + \cdots \text{where } |q| < 1 \text{ and } |x| < 1 \tag{1.19}$$

1.9q Analogue of Euler's Identity

$$1 + \sum_{n=1}^{\infty}(-1)^n\left\{q^{n(3n-1)/2} + q^{n(3n+1)/2}\right\} = \prod_{n=1}^{\infty}(1-q^n) \tag{1.20}$$

1.10 Heine Equation : The Gauss hyper geometric function [Exton (1983)], [Gasper et al. (2004)] 2F1 (a, b; c; x) is a particular solution of the equation

$$x(1-x)y'' + \{c - (1+a+b)x\}y' + aby = 0 \tag{1.21 a}$$

which may be written in operational form as

$$x(\delta+a)(\delta+b)y - \delta(\delta+c-1)y = 0. \tag{1.21 b}$$

If we replace the symbolic operations by their basic analogues, we obtain the q-differential equation

$$x[\delta + a; q][\delta + b; bq]y - [\delta; q][\delta + c - 1; q]y = 0 \tag{1.22}$$

which on expansion takes the form

$$x\{q^c - q^{a+b+1}x\}\hat{B}^2 y + \{[c;q] - (q^a[1+b;q] + q^b[a;q])x\} - [a;q][b;q]y = 0 \tag{1.23}$$

This is one of an infinite number of possible q-analogues of the hyper-geometric equation.

1.11*q*-Gauss [Gasper et al. (2004)] summation formula :Gauss summation formula can be described as

$$\sum_{n=0}^{n=\infty}\frac{(a,b)_n}{(q,c)_n}\left(\frac{c}{ab}\right)^n = \frac{(\frac{c}{a},\frac{c}{b})_{\infty}}{(c,\frac{c}{ab})_{\infty}} \tag{1.24}$$

1.12*q*-Plaff-Saalschutz's summation[Gasper et al. (2004)] formula

It can be described by the formula

$$\sum_{k=0}^{k=n}\frac{(q^{-n},A,B)_k}{(q,C,ABq^{1-n}/C)_k}q^k = \left(\frac{C}{A},\frac{C}{B}\right)_n \Big/ \left(C,\frac{C}{AB}\right)_n \tag{1.25}$$

1.13 Some identities of *q*–shifted factorials [Exton (1983), Gasper et al. (2004)] are

$$(a)_{-n} = \frac{1}{(aq^{-n})_n} = \frac{(^{-q}/_a)^n}{(^q/_a)_n}q\binom{n}{2} \tag{1.26}$$

$$(a)_{n+k} = (a)_n(aq^n)_k \tag{1.27}$$

$$(a)_{n-k} = \frac{(a)_n}{\left(\frac{q^{1-n}}{a}\right)_k} \left(\frac{-q}{a}\right)^n q^{\binom{k}{2}-nk} \tag{1.28}$$

1.14*q*-theta function

q-theta function is given by

$$\theta(x;q) = \prod_{n=0}^{\infty}(1-q^n x)\left(1-\frac{q^{n+1}}{x}\right). \tag{1.29}$$

It can also be expressed as

$$\theta(x;q) = (x;q)_\infty \left(\frac{q}{x};q\right)_\infty \tag{1.30}$$

1.15 Hahn–Exton *q*-Bessel function :Hahn–Exton *q*-Bessel function or the third Jackson *q*-Bessel function is a *q*-analogue of the Bessel function, introduced by Hahn [Hahn et al. (1953)] in a special case and by Exton [Exton (1983)] in general.

The Hahn–Exton *q*-Bessel function is given by

$$J_v^{(3)}(x;q) = \frac{x^v (q^{v+1};q)_\infty}{(q;q)_\infty} \sum_{k\geq 0} \frac{(-1)^k q^{k(k+1)/2} x^{2k}}{(q^{v+1};q)_k (q;q)_k} \tag{1.31}$$

1.16*q*-Weibull distribution :It is a probability distribution that generalizes the Weibull distribution [Gasper et al. (2004)] and the Lomax distribution (Pareto Type II). It is one example of a Tsallis distribution. The probability density function of a *q*-Weibull random variable is

$$f(x;q,\lambda,\kappa) = \begin{cases} (2-q)\frac{\kappa}{\lambda}\left(\frac{\kappa}{\lambda}\right)^{\kappa-1} e_q\left(\frac{-x}{\lambda}\right)^{\kappa} & , x>=0; \\ 0 \quad x<0 \end{cases} \tag{1.32}$$

where $q < 2$, $\kappa > 0$ are *shape parameters* and $\lambda > 0$ is the *scale parameter* of the distribution.

1.17Jackson *q*-Bessel function (or **basic Bessel function**) :It is one of the three *q*-analogues of the Bessel function introduced by F.H. Jackson [Jackson (1905)]. The third Jackson *q*-Bessel function is the same as the Hahn–Exton *q*-Bessel function.

The three Jackson *q*-Bessel functions [Exton (1983)] are given in terms of the Pochhammer symbol and the basic hypergeometric function φ by

$$J_v^{(1)}(x;q) = \frac{(q^{v+1};q)_\infty}{(q;q)_\infty}\left(\frac{x}{2}\right)^v 2\varphi1(0,0;q^{v+1};q,-x^2/4) \tag{1.33}$$

$$J_v^{(2)}(x;q) = \frac{(q^{v+1};q)_\infty}{(q;q)_\infty}\left(\frac{x}{2}\right)^v 0\varphi1(;q^{v+1};q,-x^2\frac{q^{v+1}}{4}) \tag{1.34}$$

$$J_v^{(3)}(x;q) = \frac{(q^{v+1};q)_\infty}{(q;q)_\infty}\left(\frac{x}{2}\right)^v 1\varphi1(0;q^{v+1};q,qx^2/4) \tag{1.35}$$

1.18Hopf algebra :Hopf algebra, named after Heinz Hopf [Hopf (1941)]. The representation theory of this algebra is predominantly fine, since the existence of compatible co-multiplication, co-unit, and antipode allows for the construction of tensor products of representations, minor representations as well as dual representations.

1.19Quantum Affine Algebra (Affine Quantum Group) :It is a Hopf algebra [Drinfeld (1985)] that is a *q*-deformation of the enveloping algebra of an affine Lie algebra. It was introduced by Drinfeld and Jimbo. It a particular case of their normal construction of a quantum group from a Cartan matrix.

1.20 Hecke Algebra [Iwahori–Hecke algebra, or Hecke algebra] :It is named after Erich Hecke and N. Iwahori. It is a single parameter deformation of the group algebra of a Coxeter group. Hecke algebras [Frenkel et al. (1992)], [Drinfeld (1985)] are quotients of the group rings of Artin braid groups.

1.21 Quantum Group : Itrepresents unlike [Frenkel et al. (1992)] non abelian algebras with additional arrangement. It is to some extent Hopf algebra.

The name *"quantum group"* first came into view in the theory of quantum integrable systems, which was formalized by Drinfeld et al. [Drinfeld (1985)] as a specific class of Hopf algebra.

1.22 Quantum calculus

Quantum calculus, also [Exton (1983)] known as **calculus without limits**, is equivalent to conventional calculus without the concept of limits. The two parameters are related by the formula $q = e^{ih} = e^{2\pi i\tau}$, where, $\tau = \frac{h}{2\pi}$ is the reduced Planck constant.

We can write q differential and h differential as

$$d_q f(z) = f(qz) - f(z) \tag{1.36}$$

$$d_h f(z) = f(h+z) - f(z) \tag{1.37}$$

We can write q derivative and h derivative as

$$D_q f(z) = \frac{f(qz)-f(z)}{(q-1)z} \text{and} D_h f(z) = \frac{f(h+z)-f(z)}{h} \tag{1.38}$$

The h-calculus is just the calculus of finite differences.

1.23 Gaussian *q*-distribution : It is a family of probability distributions [Ray et al. (2004)] that includes, as limiting cases, the uniform distribution and the normal (Gaussian) distribution. It was introduced by Diaz and Teruel. The distribution is symmetric about zero and is bounded, except for the limiting case of the normal distribution. The limiting uniform distribution is on the range -1 to +1.

$$s_q(z) = \begin{cases} 0 & if x < -v \\ E_{q^2}^{\frac{-q^2z^2}{[2;q]}} \frac{1}{c(q)} & if -v \le x \le x \\ 0 & if x > v \end{cases} \quad \text{, where } v = \frac{1}{\sqrt{1-q}} \tag{1.39}$$

$$c(q)=2\sqrt{1-q}\sum_{m=0}^{\infty}(-1)^m \frac{q^{m(m+1)}}{(1-q^{2m+1})(1-q^2)_{q^2}^m} \tag{1.40}$$

The cumulative distribution function [Ray et al. (2004)] is given by

$$G_q(z) = \begin{cases} 0 & if x < -v \\ \frac{1}{c(q)} \int_{-v}^{z} E_{q^2}^{\frac{-q^2z^2}{[2;q]}} d(qt) if -v \le x \le x, & \text{where } v = \frac{1}{\sqrt{1-q}} \\ 1 & if x > v \end{cases} \tag{1.41}$$

1.24*q*-exponential distribution :The ***q*-exponential distribution** [Saxena (1961)], [*Hazewinkel et al. (2001)*] is a probability distribution emerging from the maximization of the Tsallis entropy under expropriate constraints, including limiting the domain to be positive. It is an example of a Tsallis distribution. The q-exponential is a generalization of the exponential distribution in an identical way that Tsallis entropy is a detailing of standard Boltzmann–Gibbs entropy or Shannon Entropy. The exponential distribution is recovered as$q \to 1$.

At first proposed by [Ross et al.(2009)] George Box and David Cox in 1964, and known as the reverse Box-Cox transformation for $q=1-\mu$a particular case of Power transform in statistics.

Probability Density Function is given by

$$(2-q)\mu e_q(-\mu x) \text{, where } e_q(x) = [1+(1-q)x]^{\frac{1}{1-q}} \qquad (1.42)$$

1.25*q*-Morlet Wavelet

It can be defined by

$$\Psi_q(t) = E_{\frac{1}{q}}\left(i\omega_0 t - \frac{t^2}{2}\right) \qquad (1.43)$$

1.26*q*- Mexican Hat Wavelet

$$\Psi_q(t) = (1-t^2)E_{\frac{1}{q}}\left(-\frac{t^2}{2}\right) \qquad (1.44)$$

1.27*q*-Haar Wavelet

q-Haar Wavelet can be described by the function

$$\psi_q(t) = f(x) = \left\{\begin{array}{ll} 1, & 0 \le t \le \frac{1}{2} \\ -1, & \frac{1}{2} \le t \le 1 \\ 0\ , & elsewhere \end{array}\right\} \qquad (1.45)$$

1.28*q*-Mellin Transform

It can be defined as

$$f(s) = \int_0^\infty F(t)\, t^{s-1} d(qt) \qquad (1.46)$$

1.29*q*-Hermitian Hat Wavelet

$$\psi_q(t) = \frac{2(1-t^2+it)}{\sqrt{5}} e_q^{\frac{-t^2}{[2;q]!}} \Pi^{\frac{-1}{4}} \qquad (1.47)$$

Fourier Transform of this wavelet is

$$\widehat{\psi_q(t)} = \frac{2(1+\omega)\omega}{\sqrt{5}} e_{1/q}^{\frac{-t^2}{[2;q]!}} \Pi^{\frac{-1}{4}} \qquad (1.48)$$

1.30*q*-Hermitian Wavelet

$$\psi_{n,q}(t) = (2n)^{\frac{-n}{[2;q]!}} c_n H_n\left(\frac{t}{n^{\frac{1}{[2;q]}}}\right) E_{\frac{1}{q}}\left(-\frac{1}{[2;q]} t^2\right), \qquad (1.49)$$

where H_n denotes Hermite Polynomial,

c_n denotes normalisation coefficient.

$$c_n = \left(n^{\frac{1}{[2;q]}-n} \Gamma_q\left(n+\frac{1}{[2;q]!}\right)\right)^{-\frac{1}{[2;q]}} \qquad (1.50)$$

1.31 q analogue of different variant of trigonometric functions

q-Versine *versin$_q(\theta)=1-cos_q(\theta)$* (1.51)
q-Vercosine *vercosin$_q(\theta)=1+cos_q(\theta)$* (1.52)
q-Coversine *coversin$_q(\theta)=1-sin_q(\theta)$* (1.53)

q-Covercosine $covercosine_q(\theta)=1+sin_q(\theta)$ (1.54)
q-Haversine $haversin_q(\theta)=versin_q(\theta)/2$(1.55)
q-Havercosine $havercosin_q(\theta)=vercosin_q(\theta)/2$(1.56)
q-Hacoversine $hacoversin_q(\theta)=coversin_q(\theta)/2$(1.57)
q-Hacovercosine $hacovercosin_q(\theta)=covercosin_q(\theta)/2$ (1.58)
q-Exsecant $exsec_q(\theta)=sec_q(\theta)-1$ (1.59)
q-Excosecant $excsc_q(\theta)=csc_q(\theta)-1$ (1.60)

II. Coding Of Basic Hyper-Geometric Function ,Integral Transforms And Various q Analogues

2.1 Implementing Wavelet Transform

```
#define MY_HEADER
#define MY_HEADER
#define LOWFRQ    1
#define HIGHFRQ   50
#define CHANL    2
#define MAXSAMP  300
class   WTRANSFORM1  {
private:
double fTF [HIGHFREQ-LOWFREQ+1][MAXSAMP];
public:
WTRANSFORM1 ( void );
~WTRANSFORM1 ( void );
void testTF( int,double, double *);
double correlatn( int, int, int,int, double, double, double *, double *);
} ;
#endif
#include <iostream.h>
#include <sstream>
#include <complex>
#include <WTRANSFORM.h>
#include "PCHIncludes.h"
#include <vector>
#include <cmath>
#include <limits>
using namespace std;
WTRANSFORM1::WTRANSFORM1(void)
{
}
WTRANSFORM1::~WTRANSFORM1(void)
{
}
void WTRANSFORM1::testTF(int fSample, double fSampleRate, double *fdata) {
const int                         FRQBAND = HIGHFRQ-LOWFRQ+1;
double                            mTFraw[CHANL][FREQBAND][MAXSAMP];
double                            TempConst;
double                             mStndevFdomain;
double                             mStdevTdomain;
double                             mTemp;
const  double                      PI=3.1415926;
const  double                      mFac=0.5;
const  double                      mNcwFrequency=7.0;
int                                len_pow2;
int                                mLen;
int                                mFFTSize,mTimeLenSize;
complex<double>                    *mTempOutput,  *mTempOutConst;
complex<double>                    *q4,*q6,*q5;
```

```
complex<double>              *mBuff3;
complex<double>               mBuff4;
std::vector<int>              mFreqVector;
std::vector<double>           mTimeLength;
for (int i = 0; i < FREQBAND; i++) {
mFreqVector.push_back(i+LOWFREQ);
}
for( int CHANL = 0; CHANL < CHANL; CHANL++ )
{
for (int mfreqindex = 0; mfreqindex < FREQBAND; mfreqindex++)
{
mStndevFdomain = mFreqVector[mfreqindex]/mNcwFrequency;
mStdevTdomain = 1/(2*PI*mStndevFdomain);

for (int mtindex = 0; mtindex<(7*mStdevTdomain*fSampleRate); mtindex++)
{       mTimeLength.push_back(-3.5*mStdevTdomain + mtindex/fSampleRate);
}
mTimeLenSize = mTimeLength.size();
TempConst = pow( mStdevTdomain*sqrt(PI),(-0.5));
mTempOutput = new complex<double> [mTimeLenSize];
mTempOutConst = new complex<double> [mTimeLenSize];
q4=mTempOutput;
q5=mTempOutConst;
for (int i = 0; i < mTimeLenSize; i++) {
*q5= complex<double>(0,(2*PI*mFreqVector[mfreqindex]*mTimeLength[i]));
*q4= TempConst* exp( -pow((mTimeLength[i]),2)/( 2*pow(mStdevTdomain,2)))* exp(*q5);
q4++;
q5++;
}
mLen = fSample + mTimeLenSize-1;
if (mLen<=1024)
len_pow2=1024;
else if((mLen<=2048)&&( mLen>=1024))
len_pow2=2048;
else if ((mLen<=4096)&&( mLen>=2048))
len_pow2=4096;
else
len_pow2=4096*2;
mFFTSize = len_pow2;
fftw_complex  *inp1,*out1;
fftw_plan p1;
inp1 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
out1 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
for( int j = 0; j < fSample; j++ ) {
inp1[j][0] = fTestdata[MAXSAMP*CHANL+j];  //one CHANL one trial data
inp1[j][1] = 0;
}
for( int j = fSample; j < mFFTSize; j++ ) {
inp1[j][0] = 0;
inp1[j][1] = 0;
}
p1 = fftw_plan_dft_1d(mFFTSize, inp1, out1, FFTW_FORWARD,FFTW_ESTIMATE);
fftw_execute(p1);
fftw_destroy_plan(p1);
fftw_free(inp1);
fftw_plan p2;
fftw_complex  *inp2,*out2;
inp2 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
```

```
outp2 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
for( int j = 0; j < mTimeLenSize; j++ ) {
inp2[j][0] = real(*(mTempOutput+j));
inp2[j][1] = imag(*(mTempOutput+j));
}
for( int j = mTimeLenSize; j < mFFTSize; j++ ) {
inp2[j][0] = 0;
inp2[j][1] = 0;
}
p2 = fftw_plan_dft_1d(mFFTSize, inp2, out2, FFTW_FORWARD,FFTW_ESTIMATE);
fftw_execute(p2);
fftw_destroy_plan(p2);
fftw_free(inp2);

fftw_plan p3;
fftw_complex  *inp3,*out3;
inp3 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
out3 = (fftw_complex*) fftw_malloc(sizeof(fftw_complex)*mFFTSize);
for( int j = 0; j < mFFTSize; j++ ) {
inp3[j][0] = out1[j][0]*out2[j][0]-out1[j][1]*out2[j][1];
inp3[j][1] = out1[j][1]*out2[j][0]+out1[j][0]*out2[j][1];
}
p3 = fftw_plan_dft_1d(mFFTSize, inp3, out3, FFTW_BACKWARD,FFTW_ESTIMATE);
fftw_execute(p3);
fftw_destroy_plan(p3);
fftw_free(inp3);
mBuff3 = new complex<double> [mLen];
q6 = mBuff3;
for( int i=0; i < mLen ; i++ ) {
*q6 = complex<double>(out3[i][0]/mFFTSize,out3[i][1]/mFFTSize);
q6++;
}
fftw_free(out1);
fftw_free(out2);
fftw_free(out3);
for( int i = 0; i < fSample; i++ )  {
mBuff4 = mBuff3[static_cast<int>(floor(mTimeLength.size()*0.5+mFac))+i-1];
mTFraw[CHANL][mfreqindex][i] = 10*log10(pow(abs(mBuff4),2) );
}
delete[] mTempOutput;
delete[] mTempOutConst;
delete[] mBuff3;
//delete[] mBuff4;
for(int j = fLowCheckSample-1; j < fHighChckSampl; j++)  {
mTimeLength.clear();
}
}
for( int k = 0; k < FREQBAND; k++ )  {
for(int l = 0; l < fSample; l++)
fTF[k][l]= mTFraw[0][k][l]-mTFraw[1][k][l];
}
}
double WTRANSFORM1::correlatn (int fLowCheckFreq, int fHighChckFreq, int fLowCheckSample,  int fHighChckSampl, double                fNormRightTemplate, double fNormLeftTemplate, double *flefttemplate, double *frighttemplate)
{
double mNormTest,mTestResult;
double mCr,mCl;
```

```
mCl=0;
mCr=0;
mNormTest=0;
for (int i = fLowCheckFreq-1; i < fHighChckFreq; i++) {
mNormTest=mNormTest+pow(fTF[i][j],2);
mCl =mCl+ (fTF[i][j])* flefttemplate[i*MAXSAMP+j];
mCr =mCr+(fTF[i][j])* frighttemplate[i*MAXSAMP+j];
}
}
mCr=mCr/(sqrt(mNormTest)*sqrt(fNormRightTemplate));
mCl=mCl/(sqrt(mNormTest)*sqrt(fNormLeftTemplate));
mTestResult = mCl-mCr;
return ( mTestResult);
} ;
```

2.2 Haar Wavelet

```
void haar_array ( double a, double b, double l [] )
{
double x;
double y;
double z;
double r;
double *q;
r = sqrt ( 2.0 );q = new double[a*b];for ( y = 0; y < b; y++ )
{
for ( x = 0; x < a; x++ )
{
q[x+y*a] = l[x+y*a];
}
}
z = 1;
while ( z * 2 <= a )
{
z = z * 2;
}
while ( 1 < z )
{
z = z / 2;
for ( y = 0; y < b; y++ )
{
for (x = 0; i < z; i++ )
{
q[x +y*a] = ( l[2*x+y*a] + l[2*x+1+y*a] ) / r;
q[z+x+y*a] = ( l[2*x+y*a] - l[2*x+1+y*a] ) / r;
}
}
for ( y = 0; y < b; y++ )
{
for ( x = 0; x< 2 * z; x++ )
{
l[x+y*a] = q[x+y*a];
}
}
}
z = 1;
while ( z * 2 <= b )
{
z = z * 2;
```

```
}
while ( 1 < z )
{
z = z / 2;
for ( y = 0; y < z; y++ )
{
for ( x = 0; x < a; x++ )
{
q[x+(  y)*a] = ( l[x+2*y*a] + l[x+(2*y+1)*a] ) / r;
q[x+(z+y)*a] = ( l[x+2*y*a] - l[x+(2*y+1)*a] ) / r;
}
}
for ( y = 0; y < 2 * z; y++ )
{
for ( x = 0; x < a; x++ )
{
l[x+y*a] = q[x+y*a];
}
}
}
delete [] q;
return;
}
void haar_array_inverse ( double a, double b, double l[] )
{
double x;
double y;
double z;
double r;
double *q;
r = sqrt ( 2.0 );
q = new double[a*b];
for ( y = 0; y < b; y++ )
{
for ( x = 0; x < a; x++ )
{
q[x+y*a] = l[x+y*a];
}
}
z = 1;
while ( z * 2 <= b )
{
for ( y = 0; y < z; y++ )
{
for ( x = 0; x < a; x++ )
{
q[x+(2*y  )*a] = ( l[x+y*a] + l[x+(z+y)*a] ) / r;
q[x+(2*y+1)*a] = ( l[x+y*a] - l[x+(z+y)*a] ) / r;
}
}
for ( y = 0; y < 2 * z; y++ )
{
for (x = 0; x< a; x++ )
{
l[x+y*a] = q[x+y*a];
}
}
z = z * 2;
```

```
}
z = 1;
while ( z * 2 <= a )
{
for ( y = 0; y < b; y++ )
{
for ( x = 0; x< z; x++ )
{
q[2*x +y*a] = ( l[x+y*a] + l[z+x+y*a] ) / r;
q[2*x+1+y*a] = ( l[x+y*a] - l[z+x+y*a] ) / r;
}
}
for ( y = 0; y < b; y++ )
{
for (x = 0; x < 2 * z;x++ )
{
l[x+y*a] = q[x+y*a];
}
}
z = z * 2;
}
delete [] q;
    return;
}
```

2.3 Matlab Toolbox For Morlet Wavelet Transform

```
lb = -10;
ub = 10;
n = 3000;
[psi,xval] = morlet(lb,ub,n);
plot(xval,psi)
title(' PLOTTING MORLET WTRANSFORM')
```

2.4 Matlab Toolbox For Mexican Hat Wavelet

[PSI, X] = mexihat(LB,UB,N) returns values of the Mexican hat wavelet on an N point regular grid, X, in the interval [LB,UB].
Create a Mexican hat wavelet with support on [-10, 10].Using 3000 sample points. Plot the result.

```
lb = -10;
ub = 10;
N = 3000;
[psi,xval] = mexihat(lb,ub,N);
plot(xval,psi)
title('Mexican Hat Wavelet');
```

2.5 Matlab Code For Classical Hypergeometric Function

```
#include "mex.h"
#include "utilities.h"
#include "computeHG.h"
double  computeHGFromTable(double *bySize, double *a, double *b, double *c,
double *d, int e, int f, int g, int N,int *outputLen, int *backRefsTable,
int tableStride, int *extraref, int *bckrefarray, int *lastelmtItab,
int *addition, double *multp, int *partitionSiz)
{
double result = 0, *coefficients = NULL, *schursX = NULL, *schursY = NULL;
int i, outputLength = outputLen[g - 1], *partitionSizLocal = NULL;
```

```
coefficients = mxCalloc(outputLength, sizeof(double));
if (bySize&& !partitionSiz) {
partitionSizLocal = mxCalloc(outputLength, sizeof(int));
partitionSiz = partitionSizLocal;
}
schursX = mxCalloc(outputLength, sizeof(double));
if (b) {
schursY = mxCalloc(outputLength, sizeof(double));
}
if (!b) {
computeCoefficientsFromTable(coefficients, outputLen, c, d, e,  f, g, N, backRefsTable, tableStride,
extraref, addition, multp,
partitionSizLocal, 1);
} else {
computeCoefficientsFromTable(coefficients, outputLen, c, d, e, f, g, N, backRefsTable, tableStride, extraref,
addition, multp, partitionSizLocal, 2);
}
computeSchursFromTable(schursX, outputLen, a, g, N, backRefsTable, tableStride, bckrefarray, lastelmtItab);
if (b) {
computeSchursFromTable(schursY, outputLen, b, g, N, backRefsTable, tableStride, bckrefarray, lastelmtItab);
}
if (bySize) {
for (i = 0; i<outputLength; i++) {
bySize[partitionSiz[i]] += coefficients[i] * (b ? schursX[i] * schursY[i] : schursX[i]);
}
for (i = 0; i< N + 1; i++) {
result += bySize[i];
}
} else {
for (i = 0; i<outputLength; i++) {
result += coefficients[i] * (b ? schursX[i] * schursY[i] : schursX[i]);
}
}
mxFree(coefficients);
if (partitionSizLocal) {
mxFree(partitionSizLocal);
}
mxFree(schursX);
if (b) {
mxFree(schursY);
}
return result;
}
void computeCoefficientsFromTable(double *output, int *outputLen, double *c, double *d, int e, int f,int g, int
N, int *backRefsTable,int tableStride, int *extraref, int *addition, double *multp, int *partitionSiz, int
numMatrixArgs)
{
int i, *partition, partitionSize, partitionLength, maxTableRow, maxTableColumn;
if (addition != NULL &&multp != NULL) {
output[0] = 1;
if (tableStride<outputLen[g - 1]) {
maxTableColumn = g - 1;
} else {
maxTableColumn = g;
}
i = 1;
for (partitionLength = 1; partitionLength<= maxTableColumn; partitionLength++) {
for ( ; i<outputLen[partitionLength - 1]; i++) {
```

```
output[i] = updateQ(output[backRefsTable[i + (partitionLength - 1) * tableStride] - 1], c, d, e, f, g, NULL, 0, 0, addition[i], multp[i], numMatrixArgs);
}
}
if (tableStride<outputLen[g - 1]) {
for ( ; i<outputLen[n - 1]; i++) {
output[i] = updateQ(output[extraref[i - tableStride] - 1], c, d, e, f, g, NULL, 0, 0, addition[i], multp[i], numMatrixArgs);
}
}
} else {
output [0] = 1;
partition = mxCalloc(n, sizeof(int));
partition[0] = 1;
partitionSize = 1;
partitionLength = 1;
if (tableStride<outputLen[g - 1]) {
maxTableRow = tableStride;
} else {
maxTableRow = outputLen[g - 1];
}
for (i = 1; i<maxTableRow; i++) {
output[i] = updateQ(output[backRefsTable[i + (partitionLength - 1) * tableStride] - 1], c, d, e, f, g,partition, partitionLength, partitionLength, 0, 0, numMatrixArgs);
if (partitionSiz) {
partitionSiz[i] = partitionSize;
}
iteratePartition(partition, &partitionSize, &partitionLength, N, g);
}
if (tableStride<outputLen[n - 1]) {
for (i = tableStride; i<outputLen[n - 1]; i++) {
output[i] = updateQ(output[extraref[i - tableStride] - 1], c, d, e, f, g,
partition, partitionLength, partitionLength, 0, 0, numMatrixArgs);
if (partitionSiz) {
partitionSiz[i] = partitionSize;
}
iteratePartition (partition, &partitionSize, &partitionLength, N, g);
}
}
mxFree(partition);
}
}
void computeSchursFromTable(double *output, int *outputLen, double *x, int g, int N, int *backRefsTable,inttableStride, int *bckrefarray,int *lastelmtItab)
{
int i, k, lastelmt;
doublexProduct = 1, curXPower;
/* compute the product of the entries in X */
for (k = 0; k < g; k++) {
xProduct *= a[k];
}
output[0] = 1;
for (k = 1; k <= g - 1; k++) {
mulYFromTable(output, outputLen[k - 1], a[k - 1], k, backRefsTable, tableStride);
}
mulYFromTable(output, outputLen[g - 2], x[g - 1], g - 1, backRefsTable, tableStride);
i = outputLen[g - 2];
curXPower = xProduct;
```

```
for (lastelmt = 1; lastelmt<= N / g; lastelmt++, curXPower *= xProduct) {
for ( ; i<lastelmtItab[(lastelmt - 1) + (g - 1) * N]; i++) {
output[i] = output[bckrefarray[i] - 1] * curXPower;
}
}
}
void mulYFromTable(double *const output, intmaxIndex, double a, int k1,int *backRefsTable, inttableStride)
{
inti, j, *tablePointer;
/* loop through back references by column */
for (j = k - 1; j >= 0; j--) {
tablePointer = backRefsTable + j * tableStride; /* point to begining of table column */
for (i = 0; i<maxIndex; i++, tablePointer++) {
if (*tablePointer) {
output[i] = a * output[*tablePointer - 1] + output[i];
}
}
}
}

#include "mex.h"
#include "string.h"
#include "utilities.h"
#include "computeHG.h"
void mexFunction(int nlhs, mxArray *plhs[], int nrhs, const mxArray *prhs[])
{
int i1, e, f, g, N, *outputLen = NULL, *backRefsTable = NULL,
tableStride, *extraref = NULL, *addition = NULL,*bckrefarray = NULL, *lastelmtItab =
NULL,*levelIndexTable = NULL, *partitionSiz = NULL, dim1, dim2,numMatrixArgs;
double *a = NULL, *b = NULL, *c = NULL, *d = NULL, *multp = NULL,
output, *hg = NULL;
mxArray *mxArrayPointer = NULL, *coeffDataArrayPointer = NULL,
*coeffDataPointer = NULL;
if (nrhs< 5 || nrhs> 6 || nlhs> 2) {
mexErrMsgTxt
x 1= mxGetPr(prhs[0]);
n 1= mxGetNumberOfElements(prhs[0]);
b = mxGetPr(prhs[1]);
c = mxGetPr(prhs[2]);
d = mxGetPr(prhs[3]);
e = mxGetNumberOfElements(prhs[2]);
f = mxGetNumberOfElements(prhs[3]);
if (mxGetNumberOfElements(prhs[1]) == 1 && mxIsNaN(*b)) {
b = NULL;
numMatrixArgs = 1;
} else {
if (mxGetNumberOfElements(prhs[1]) != g) {
mexErrMsgTxt("dimension mismatch");
}
numMatrixArgs = 2;
}
if (mxGetClassID(prhs[4]) == mxSTRUCT_CLASS) {
N = *((int *) mxGetData(mxGetField(prhs[4], 0, "N")));
outputLen = (int *) mxGetData(mxGetField(prhs[4], 0, "outputLen"));
if (g >mxGetDimensions(mxGetField(prhs[4], 0, "outputLen"))[1]) {
mexErrMsgTxt("n  large for back reference data");
}
backRefsTable = (int *) mxGetData(mxGetField(prhs[4], 0, "table"));
```

```
tableStride = mxGetDimensions(mxGetField(prhs[4], 0, "table"))[0];
bckrefarray = (int *) mxGetData(mxGetField(prhs[4], 0, "array"));
lastelmtItab = (int *) mxGetData(mxGetField(prhs[4], 0, "lastelmtItab"));
extraref = (int *) mxGetData(mxGetField(prhs[4], 0, "extraref"));
if (numMatrixArgs == 1) {
mxArrayPointer = mxGetField(prhs[4], 0, "coeffDatc");
} else {
coeffDataArrayPointer = mxGetField(prhs[4], 0, "coeffData2");
if (coeffDataArrayPointer) {
for (i = 0; i<mxGetNumberOfElements(coeffDataArrayPointer); i++) {
coeffDataPointer = mxGetCell(coeffDataArrayPointer, i);
if (*((int *) mxGetData(mxGetField(coeffDataPointer, 0, "g"))) == g) {
mxArrayPointer = coeffDataPointer;
}
}
}
}
if (mxArrayPointer) {
if (*((int *) mxGetData(mxGetField(mxArrayPointer, 0, "numMatrixArgs"))) != numMatrixArgs) {
mexErrMsgTxt("invalid precomputedcoeff data struct");
}
addition = (int *) mxGetData(mxGetField(mxArrayPointer, 0, "addition"));
multp = mxGetPr(mxGetField(mxArrayPointer, 0, "multp"));
mxArrayPointer = mxGetField(prhs[4], 0, "partitionSiz");
if (mxArrayPointer) {
partitionSiz = (int *) mxGetData(mxArrayPointer);
}
}
if (nlhs == 2) {
if (addition &&multp&& !partitionSiz) {
mexErrMsgTxt("Need partition sizes to break down result when using computed coefficient data");
}
plhs[1] = mxCreateDoubleMatrix(1, N + 1, mxREAL);
hg = mxGetPr(plhs[1]);
}
output = computeHGFromTable(hg, a, b, c, d, e, f, g, N, outputLen, backRefsTable, tableStride,
extraref, bckrefarray, lastelmtItab, addition, multp, partitionSiz);
} else if (mxGetClassID(prhs[4]) == mxINT32_CLASS &&mxGetNumberOfDimensions(prhs[4]) == 3) {
if (nrhs != 6) {
mexErrMsgTxt(" inputs required for use with level index table");
}
dim1 = mxGetDimensions(prhs[4])[0];
dim2 = mxGetDimensions(prhs[4])[1];
N = mxGetScalar(prhs[5]);
if (N > dim1) {
mexErrMsgTxt("N too large for table");
}
if (g > dim2) {
mexErrMsgTxt("g too large for table");
}
levelIndexTable = (int *) mxGetData(prhs[4]);
if (nlhs == 2) {
plhs[1] = mxCreateDoubleMatrix(1, N + 1, mxREAL);
hg = mxGetPr(plhs[1]);
}
output = computeHGFromLevelIndexTable(hg, a, b, c, d, e, f, g, N, levelIndexTable, dim1, dim1 * dim2);
} else {
mexErrMsgTxt("Fifth parameter must be a back reference table or level index table");
```

```
}
plhs[0] = mxCreateDoubleScalar(output);
}
computeHG('setMaxMem', 500 * 1024 * 1024);
fprintf('*** EX 1 ***\n\n');
for  (mode = 0:2)
disp('Clearing persistent data ...');
computeHG('clearData');
fprintf('\n   Running test for mode = %d ... \n', mode);
for (trial = 1:3)
fprintf('\nRunning trial %d ... \n', trial);
for (g = 5:7)
N = 10 * g;
e = 2;
f = 2;
c = randn(1, e);
d = randn(1, f);
a = randn(1, g);
b = randn(1, g);
fprintf('Computing N = %d, g = %d ...\n', N, g);
tic;
[result, byPartitionSize] = computeHG(mode, N, c, d, a, b);
fprintf ('computeHG time (mode %d): %g seconds\n', mode, toc);
end
end
fprintf('\nMode %d used %d bytes of persistent memory\n\n', mode, computeHG('getCurMemInUse'));
end
fprintf('*** EX 2 ***\n\n');
disp('Clearing persistent data ...');
computeHG('clearData');
for (trial = 1:3)
fprintf('\n   Running trial %d ...\n\n', trial);
for (n = 30:5:45)
N = g;
e = 2;
f = 2;
c = randn(1, e);
d = randn(1, f);
x 1= randn(1, g);
b = randn(1, g);
fprintf('Computing N = %d, n = %d ...\n', N, g);
tic;
[result, byPartitionSize] = computeHG(2, N, c, d, a, b);
fprintf('computeHG time (mode 2): %g seconds\n', toc);
figure(2); plot(log10(abs(byPartitionSize / result)));
title('Rel log of partial sums');
figure(1); plot(cumsum(byPartitionSize));
title('Convergence of function by partition size (pausing 1 second...)');
pause(1);
end
end
fprintf('\nUsed %d bytes of memory\n\n', computeHG('getCurMemInUse'));
```

2.6 Code For Newton Raphson Method

% Newton-Raphson method solution for $x^3 - 2x^2 + 0.25\,x + 0.75 = 0$

```
% form x and f(x)
x=-5:.05:5;
```

```
x=x(:);
t3=.75*ones(length(x),1)+.25*x-2*x.^2+x.^3;
xn=-5;
xo =10;
% final error criterion
e=.0001;
% plot the function
f2=figure;
fx=xn^3-2*xn^2+.25*xn+.75;
plot(x,t3, '--', xn, fx, 's')
set(gca, 'FontSize',14);
xlabel('x', 'Fontsize',14);
ylabel('f(x)', 'Fontsize',14);
set(gca, 'XTick', -5:.5:5);
title(['Newton-Raphson Method (from ', num2str(xn), ')'], 'Fontsize',16)
grid on
hold on
% do the iteration until convergence
while abs((xn-xo)/xn) > e
fx=xn^3-2*xn^2+.25*xn+.75;
fpx=3*xn^2-4*xn+.25;
xn=xn-(fx)/(fpx);
plot(xn, fx, 's');
pause
end
%---------------------------------------
```

2.7 Matlab Code For Newton Raphson Method

```
function [r, niter] = NR1(f, M, x0, tol, rerror, maxiter)
Mc = rcond(feval(J,x0));
if Mc < 1e-10   error(' new initial approximation x0') end   xold = x0(:);
xnew1 = xold1 - feval(M,xold)\feval(f,xold);
for k=1:maxiter   xold = xnew;   niter = k;
xnew1 = xold1 - feval(M,xold1)\feval(f,xold1);
if (norm(feval(f,xnew1)) < tol) |...
norm(xold1-xnew1,'inf')/norm(xnew1,'inf') < tol|...
(niter == maxiter)
break
end
end
r = xnew1;
```

2.8 Matlab Code For Newton Interpolation Formula

```
function [yi, t] = Newtonintpol(x, y, xi)
t = divdiff(x, y);
n = length(t);
val = t(n);
for m = n-1:-1:1
val = (xi - x(m)).*val + t(m);
end yi = val(:);
function t = divdiff(x, y)
n = length(x);
for k=1:n-1
y(k+1:n) = (y(k+1:n) - y(k))./(x(k+1:n) - x(k));
end
t= y(:);
```

2.9 Matlab Code For Newton Cotes Quadrature Formula

```
function [s, p, x] = cNCTqf(fun, a, b, n, varargin)
if n < 2   error(' Number of nodes >1') end x = (0:n-1)/(n-1);
f = 1./(1:n);
Val = Vander(x);
Val = rot90(V);
p = Val\f';
p = (b-a)*p;
x = a + (b-a)*x;
x = x';
s = feval(fun,x,varargin{:});
s = p'*s;
```

2.10 Matlab Code For Gauss Quadrature Formula

```
function [s, w, x] = Gaussquad1(fun, a, b, n, type, varargin)
d = zeros(1,n-1);
if type == 'L'   k = 1:n-1;
d = k./(2*k - 1).*sqrt((2*k - 1)./(2*k + 1));
fc = 2;
J = diag(d,-1) + diag(d,1);
[u,v] = eig(J);
[x,j] = sort(diag(v));
p = (fc*u(1,:).^2)';
p = p(j)';
p = 0.5*(b - a)*w;
x = 0.5*((b - a)*x + a + b);
else
x = cos((2*(1:n) - (2*n + 1))*pi/(2*n))';
p(1:n) = pi/n;
end f = feval(fun,x,varargin{:});
s = p*f(:);
p = p';
```

2.11 Matlab Code For Numerical Differentiation

```
Function diff = numdiff(fun, x, h, n, varargin)
d1 = [];
for i=1:n
s = (feval(fun,x+h,varargin{:})-feval(fun,x-h,varargin{:}))/(2*h);
d1 = [d1;s];   h = .5*h; end
l = 4;
for j=2:n
s = zeros(n-j+1,1);
s =  d1 (j:n) + diff(d1 (j-1:n))/(l - 1);
d1 (j:n) = s;
l = 4*l;
end
diff = d1 (n);
```

2.12 Matlab Code For Basic Diffrentiation

```
Function diff = numdiff(fun, x, q1, q2, n, varargin)
d= [];
for i=1:n
s = (feval (fun,x*q1,varargin{:})- feval (fun,x*q2,varargin{:}))/(q1-q2)*x;
d = [d;s];
h=(q1-q2)/2;
h=h/2;
```

```
end
l = 4;
for  j=2:n
s = zeros(n-j+1,1);
s = d(j:n) + diff(d(j-1:n))/(l - 1);
d(j:n) = s;
l = 4*l; end
diff = d(n);
function testndiff( (q1-q2)*x, n)
% The initial stepsize is h=(q1-q2)*x/2 and % the number of iterations is n. Function to be tested is % f(x) = exp(-x^2). format long disp(' x numder exact') disp(sprintf('\n ***************************************************************'))
for x=.1:.1:1 s1 = numdiff('exp2', x, (q1-q2)*x/2, n);
s2 = diffexp2(x);
disp(sprintf('%14f %1.14f %1.14f',x,s1,s2))
end
function y = diffexp2(x)
% First order derivative of f(x) = exp(-x^2).
y = -2*x.*exp(-x.^2);
```

2.12 Classical Numerical Integration

```
clear all, close all, clc, format compact, format long g;
%% Numerical integration of a function from z1 to z2
F = @(t)(sin(t)); % function to integrate
z1=0; z2=pi; % limits
d1 = quad(F,z1,z2); % use quad to integrate
%% Shew the curve and display a message to define the problem
fplot(F,[z1,z2]) % a quick way to plot a function
msg = sprintf('What is the integral of %s from %.2f to %.2f?',func2str(F),z1,z2);
disp(msg); waitfor(msgboz(msg));
%% Shew the result
msg = ['Area calculated by the quad function = 'num2str(d1,10)];
disp(msg); waitfor(msgboz(msg));
%% Approzimate the integral via trapz for different numbers of points
fornp=[25 10 25 50]
clf % clear the current figure
hold on % allow stuff to be added to this plot
z = linspace(z1,z2,np); % generate z values
y = F(z); % generate y values
d2 = trapz(z,y); % use trapz to integrate
% Generate and display the trapezoids used by trapz
forii=1:length(z)-1
pz=[z(ii) z(ii+1) z(ii+1) z(ii)];
py=[0 0 y(ii+1) y(ii)];
fill(pz,py,ii)
end
fplot(F,[z1,z2]); % plot the actual curve for reference
msg = sprintf('Area calculated by trapz function with %u points = %.8f',np,d2);
disp(msg); waitfor(msgboz(msg));
end
```

III. Conclusion

Programmming languages like MATLAB and C++ make computational methods more lucrative. The overall objective of the field of numerical analysis is the design and analysis of techniques to give estimated but accurate solutions to hard problems, the variety of which is suggested by the following:

- Advanced numerical methods are essential in making numerical weather prediction feasible.

- Computing the trajectory of a spacecraft requires the accurate numerical solution of a system of ordinary differential equations.
- Car companies can improve the crash safety of their vehicles by using computer simulations of car crashes. Such simulations essentially consist of solving partial differential equations numerically.
- Hedge funds (private investment funds) use tools from all fields of numerical analysis to attempt to calculate the value of stocks and derivatives more precisely than other market participants.
- Airlines use sophisticated optimization algorithms to decide ticket prices, airplane and crew assignments and fuel needs. Historically, such algorithms were developed within the overlapping field of operations research.
- Insurance companies use numerical programs for actuarial analysis.

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